Worker selection and efficiency*

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Abstract

We study a model where a manager repeatedly selects one worker from a group of homogeneous workers to perform a task. We characterize the largest set of parameters under which an equilibrium achieving efficient worker performance exists. We then show that this is the set of parameters given which the following manager's strategy constitutes an efficient equilibrium: the manager cyclically orders all workers and if the task is undesirable (resp., desirable), a worker is selected until good (resp., bad) performance, after which the manager randomizes between reselecting him and moving to the next worker; the reselection probability is set to be as high as effort incentives permit. Our findings extend to repeated selection of multiple workers.

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1 Introduction

Worker performance is crucial to organizational success, yet motivating workers remains a persistent challenge for managers, particularly middle managers in large organizations who lack the authority or resources to deploy monetary incentives. Managers therefore often need to rely on other levers. This paper focuses on one such lever: the dynamic allocation of a task. We study how a manager can structure task assignments among a group of homogeneous workers over time to improve worker performance.

For concreteness, we begin by considering settings where the task is inherently undesirable. Consider, for example, the burden of Kitchen Patrol duty in a typical military unit. Soldiers dislike cleaning pots and peeling potatoes, but this work must be done, and done well, for the camp to function. Similarly, in a hospital unit, employees dislike night shifts which someone must take. Formally, we study a model of dynamic moral hazard in which, in each period, a manager must select a worker from a group of homogeneous workers to perform a task. In each period, the selected worker privately chooses whether to exert effort or to shirk; this action produces a noisy public output, which is either good or bad. Each worker prefers to rest, i.e., to not be selected. Conditional on being selected, each worker prefers shirking to exerting effort. Efficiency calls for workers to exert effort whenever selected. The manager prefers good outputs to bad outputs. Our interest is in examining the manager's strategy, which we interpret as a selection rule, that constitutes an efficient equilibrium.

In our model, effort incentives are intrinsically dynamic. To motivate effort from a selected worker, the manager must, in expectation, reward the worker with future rest time and punish them with reselection. To this end, the manager must resolve the following tension: longer rest periods for one worker strengthen his incentive to exert effort, yet simultaneously require selecting other workers more often, thereby undermining their effort incentives.

Our first main result characterizes, for each number of workers, the largest set of parameters under which an efficient equilibrium exists. This set admits intuitive comparative statics. It expands as the workers become more patient, output becomes a more informative signal of effort, the gain from shirking falls, or the benefit from resting increases. It also expands as the number of workers increases: utilizing more workers allows the manager to reward each worker with longer expected rest time and strengthen effort incentives.

In proving this result, we address an open problem in the literature on repeated principal-agent models without transfers when there are more than two agents. In the case of only two workers, the set of efficient perfect public equilibrium (PPE) payoffs of the workers is an interval. Consequently, extreme payoffs in the set corresponding to rewards and punishments are readily identified, facilitating the characterization of the largest set of parameters attaining efficiency in equilibrium. With more than two workers, this approach does not apply because the topological structure of the efficient equilibrium payoff set is not well understood (see, e.g., De Clippel, Eliaz, Fershtman and Rozen, 2021). Our analysis offers an approach that circumvents this issue: we simultaneously solve for the boundary of the largest set of parameters attaining equilibrium efficiency as well as necessary conditions on the efficient PPE payoff vectors when parameters lie on the boundary. The boundary then pins down the largest set of parameters attaining equilibrium efficiency.

Our second main result presents an equilibrium that is efficient whenever some efficient equilibrium exists, namely whenever the parameters lie in the set characterized in our first main result. In this equilibrium, the manager plays an inertial selection rule: he cyclically orders all workers, and a selected worker continues to be selected until he produces a good output, after which the manager randomizes between reselecting him and selecting the next worker in the order. It is inertial in the sense that the reselection probability is set to be as high as the worker's effort incentives permit. Consequently, the largest set of parameters under which an efficient equilibrium exists is the set of parameters under which inertial rotation constitutes an efficient equilibrium.

Intuitively, this equilibrium sustains the strongest possible effort incentives for all workers by maximally separating the reward for success and the penalty for failure, subject to efficiency. When a worker is selected, he receives the lowest continuation payoff attainable in any efficient equilibrium: a bad output leads to immediate reselection, while a good output results in rest with a probability set so low that the worker's effort constraint binds. Conversely, conditional on being granted rest right after delivering good performance, the worker receives the highest continuation payoff attainable in any efficient equilibrium. Any alternative equilibrium play promising this worker an even higher payoff would necessarily require granting more rest time, which would inevitably force a reduction in some other worker's rest, thereby weakening this latter worker's effort incentives. This alternative therefore falls short of efficiency over a range of parameters where efficiency is attainable through inertial rotation in equilibrium.

We examine two extensions. First, we consider a version of our model where the task is desirable. Here, workers prefer to be selected, although they retain an incentive to shirk once selected. For instance, a worker prefers to be chosen for an expatriate position in a desirable city or country; once assigned, this worker is tempted to cut corners because effort is costly. The manager's challenge is then to use selection as a reward while preventing selected workers from coasting. Through analogous arguments, we derive the largest set of parameters given which an efficient equilibrium exists, and that this set is the set of parameters under which inertial rotation constitutes an efficient equilibrium. The key distinction from our baseline model is that inertia in this case arises from maximizing reward rather than punishment.

We next consider a setting where the manager must select a team consisting of multiple workers in each period. We show that our results extend naturally in both cases of desirable and undesirable tasks. This extension introduces an additional challenge that the number of available resting spots may be smaller than the team size. We show that in this event, to maximize effort incentives for all team members, the manager ensures that, conditional on some team member being selected to rest after a good (for an undesirable task) or bad (for a desirable task) output, every team member is selected to rest with equal probability. This contrasts with Winter's (2004) famous finding in moral-hazard-in-team problems where optimal incentive mechanisms assign nonsymmetric rewards even when agents are homogeneous.

Our results offer a complementary perspective on the widespread practice of work shifts or rotations rather than assigning one fixed person to perform specific tasks in organizations. While the conventional wisdom that work shifts signal fairness among workers and in turn improve their job satisfaction,[1] our results suggest that such work shifts can be designed in a way that improve effort incentives. To fix ideas, consider time-based, fixed-term shifts that are easily implementable in practice. Our model suggests that such fixed shifts are often inefficient because a worker who just failed is still immediately relieved, which weakens the punishment for underperformance in the case of undesirable tasks and weakens the reward for good performance in the case of desirable tasks. Our results therefore shed light on why certain high-stakes operations, such as in control rooms or trading floors, might bypass strict time-based schedules in favor of performance-based rotations.

[1]See, e.g., Knauth and Hornberger (2003) and McHugh, Farley and Rivera (2020).

The idea that nonmonetary instruments help improve worker performance is not new; our contribution is to introduce and study worker selection as one such instrument.[2] Our paper therefore contributes primarily to the literature on dynamic allocation relationships between a principal and many workers. The closest paper is De Clippel et al. (2021), who study dynamic adverse selection without transfers. In their model, workers have private types and take observable actions, whereas in ours workers have no private types and choose hidden actions. In the case of two workers, they characterize a selection rule that attains efficiency whenever some rule can.[3] With more workers, they study a specific class of selection rules that achieves efficiency only for some parameters. Because these rules require symmetric treatment of currently unselected workers, they fall short of efficiency in cases where inertial rotation can achieve it in our model. To our knowledge, this paper is the first to characterize the full set of parameters enabling efficiency and derive efficient allocation dynamics when there are more than two workers and no transfers, offering an approach to circumvent an unaddressed challenge as described above.

Board (2011) and Andrews and Barron (2016) study dynamic allocation relationships between a principal and multiple workers subject to moral hazard in the presence of monetary transfers. In their settings, allocation dynamics are shaped by transfers, whereas in our model they arise solely from the need to provide dynamic incentives because transfers are absent. As a result, their dynamics differ fundamentally from ours. In Board (2011), the dynamics are driven by the principal's varying costs of investing in different workers over time, which causes the principal to have an insider bias to invest in previously invested workers even when their costs exceed others. In our model, the reselection inertia can be interpreted as an outsider bias in the case of undesirable tasks and an insider bias in the case of desirable tasks, but they emerge from pure moral hazard concerns. In Andrews and Barron (2016), dynamics are driven by the workers' varying idiosyncratic productivities, leading the principal to select

[2]Most existing work focuses on incentive instruments for a single worker, unlike us. Examples include delegation (Li, Matouschek and Powell, 2017; Lipnowski and Ramos, 2020), managerial attention (Halac and Prat, 2016), feedback design (Fong and Li, 2016; Ely, Georgiadis and Rayo, 2025), public ratings (Ekmekci, 2011; Hörner and Lambert, 2021; Vong, 2025b) and mediation (Vong, 2025a).

[3]This is also related to Athey and Bagwell (2001), who study how two firms with private cost information sustain first-best collusion in a repeated Bertrand model. Like in our inertial rotation equilibrium in the two-worker case, they show that the firms achieve the first best by asymmetrically splitting market shares over time, using these splits as rewards and punishments. They establish sufficient conditions for this to constitute an equilibrium under some discount factor.

the worker who most recently produced a good output among the most productive workers in each period. These dynamics do not apply to our model where all workers are equally productive, because then a worker who is selected and produces a good output would then always be selected, disrupting effort incentives.

2 Model

Time $t = 0, 1, \ldots$ is discrete and horizon is infinite. There is a manager and a set of homogeneous workers $N := \{1, \ldots, n\}$, where $n \geq 1$.

In each period t, the manager selects one of the workers.[4] The workers observe whom among them is selected. The selected worker privately chooses whether to exert effort or to shirk. Effort yields a good output $\bar{y}$ with probability $p \in (0, 1)$ and a bad output $\underline{y}$ otherwise, whereas shirking yields a good output $\bar{y}$ with probability $q \in (0, p)$ and a bad output otherwise. This output is publicly observable. Unselected workers have no moves. Period $t + 1$ then unfolds.

In each period, each unselected worker receives payoff $r > 0$, capturing his gain from resting. The selected worker gets payoff normalized to zero if he exerts effort and payoff $s > 0$ if he shirks. Therefore, workers dislike being selected and upon being selected, they prefer shirking to exerting effort.[5] *A priori*, we impose no restriction on each worker's preference between resting and shirking.[6] On the other hand, we assume $v(\bar{y}) > v(\underline{y})$, so that the manager prefers good outputs to bad outputs. We also assume

$$pv(\bar{y}) + (1 - p)v(\underline{y}) > qv(\bar{y}) + (1 - q)v(\underline{y}) + s, \quad (1)$$

so that effort is efficient.[7]

A public history in each period t, denoted by h_t, is an element in $(N \times \{\bar{y}, \underline{y}\})^t$, consisting of the identities of the selected workers and their outputs, with $h_0 := \varnothing$. The manager's strategy is a collection $f \equiv (f_t)_{t=0}^{\infty}$, where $f_t(h_t) \in \Delta(N)$ specifies a distribution over workers

[4]This rules out the possibility that the manager selects no worker. This restriction is innocuous for our results, so long as the manager must select a worker in each period to the extent that efficiency is concerned.

[5]In Section 7.1, we consider an alternative version of our model where workers prefer to be selected.

[6]It is nonetheless an implication of our results that $r > s$ is a necessary condition for efficiency.

[7]It is not important for our results that the manager's preference over outputs aligns with efficiency.

from which the currently selected worker is drawn at history h_t. Worker i's strategy is a collection $\sigma^i \equiv (\sigma_t^i)_{t=0}^{\infty}$, where $\sigma_t^i(h_t) \in [0,1]$ specifies worker i's probability of exerting effort in period t conditional on being selected at history h_t. This definition of a worker's history omits its past private actions; this restriction helps simplify the exposition and is innocuous for our results, because monitoring has a product structure.[8]

The manager and the workers share a common discount factor $\delta \in (0,1)$.[9] To formally define their payoffs in the repeated game, we specify some additional notation. In each period t, let $b_t^i = 1$ denote the event that worker i is selected and let $b_t^i = 0$ denote the event that he is not selected. Let $a_t^i := \{0,1\}$ denote worker i's action conditional on being selected so that $a_t^i = 1$ represents effort and $a_t^i = 0$ represents shirking. Each worker i's realized payoff is

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t\Big[b_t^i(1-a_t^i)s + (1-b_t^i)r\Big].$$

Let y_t denote the realized output in period t. The manager's realized payoff is

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t v(y_t).$$

The solution concept we use is public perfect equilibrium, henceforth equilibrium. An equilibrium exists in which the manager selects each worker with equal probability in each period irrespective of the history of play, and each worker shirks upon being selected. An equilibrium is efficient if each worker exerts effort whenever he is selected. In an efficient equilibrium, the manager attains her first-best payoff $pv(\bar{y}) + (1-p)v(\underline{y})$. We interpret the manager's strategy as a selection rule, and refer to the manager's strategy in any efficient equilibrium as an efficient selection rule. Observe that in any equilibrium, only the manager has observable deviations. Without loss of generality, we assume that following any off-path history, the manager persistently selects worker 1 who then shirks, and hereafter we omit mentioning off-path equilibrium behavior for conciseness.

In any efficient equilibrium, for each worker, being selected (and exerting effort) is a

[8]Fudenberg and Levine (1994) show that in repeated games with imperfect public product-structure monitoring, the set of sequential equilibrium payoffs and the set of public perfect equilibrium payoffs coincide. Therefore, to the extent that we are concerned with efficient equilibrium payoff vectors, there is no loss of generality in restricting attention to public strategies (and using public perfect equilibrium as the solution concept).

[9]Our results extend if the manager has a different discount factor.

punishment that yields a low immediate payoff, while not being selected is a reward that yields a high immediate payoff. The manager uses these punishments and rewards over time to provide effort incentives. Her strategy is flexible, and so her selection in each period may depend on past selections and past outputs in arbitrary ways.

Given any number n of workers, let

$$\Omega(n) := \{(p, q, r, s, \delta) \in (0,1)^2 \times \mathbf{R}^2_{++} \times (0,1) : p > q\} \tag{2}$$

be the set of parameters other than n specified above, with typical element ω. Let $\Omega^*(n) \subseteq \Omega(n)$ be the (largest) set of such parameters given which an efficient equilibrium exists. That is, if $\hat{\Omega}(n)$ is a set of parameters such that an efficient equilibrium exists, then $\hat{\Omega}(n) \subseteq \Omega^*(n)$.

We close this section with a useful preliminary result concerning each worker's effort incentive in any efficient equilibrium. In any equilibrium (σ, f), let

$$U^i_{\sigma,f}(h_t) := \mathbf{E}_{\sigma,f}\left[(1-\delta)\sum_{\tau=t}^{\infty}\delta^{t-\tau}\left[b^i_\tau(1-a^i_\tau)s + (1-b^i_\tau)r\right]\middle| h_t\right]$$

denote worker i's continuation payoff at history h_t, where the expectation is taken with respect to the probability distribution over outcomes induced by (σ, f). Let $h_t iy$ denote the public history that is a concatenation of history h_t followed by a selection of worker i who then produces output y. When there is no risk of ambiguity, we write $U^i_{\sigma,f}$ simply as U^i.

Lemma 1. *In any efficient equilibrium (σ, f), at any history h_t where worker i is selected with positive probability,*

$$U^i(h_t i\bar{y}) - U^i(h_t i\underline{y}) \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{3}$$

If the interaction were one-shot, then a selected worker has a strict incentive to shirk. In our model, effort incentives are dynamic and a selected worker is motivated to exert effort only if his continuation payoff upon a good output is sufficiently higher than that upon a bad output. The incentive constraint (3) makes precise that their difference must be at least the wedge $(1-\delta)s/[\delta(p-q)]$. Naturally, this wedge is lower when the worker has a higher discount factor δ so that he is more patient, when he has a lower shirking gain s, or when a high output becomes a stronger signal of effort relative to shirking, i.e., when $p - q$ is higher.

Lemma 1 implies that if an efficient equilibrium exists, then there are at least two workers. Indeed, if there were only one worker, then the same worker must be selected in every period. In any putative efficient equilibrium, at every history, this worker's continuation payoff upon producing any output is zero, contradicting (3). Accordingly, hereafter, we assume that there are at least two workers, i.e., $n > 1$.

3 Selection rules

In this section, we illustrate some simple selection rules to motivate our main results. As will be described formally in the later sections, two desirable properties of selection rules are inertia and rotation. Here, we first illustrate inertia and then turn to rotation.

3.1 Inertia

Suppose for now that there are only two workers. Consider a selection rule in which initially worker 1 is selected. Then, in each period, the currently selected worker is reselected in the next period after a bad output, and the other worker is selected in the next period after a good output. If this selection rule is efficient, then, by writing U_S as the selected worker's continuation payoff and U_R as the unselected worker's continuation payoff in each period in the corresponding efficient equilibrium,[10] the payoffs (U_S, U_R) solve the system of promise-keeping constraints

$$U_S = \delta\left(pU_R + (1-p)U_S\right),$$
$$U_R = (1-\delta)r + \delta\left(pU_S + (1-p)U_R\right).$$

Writing U_S and U_R as $U_S(p, r, \delta)$ and $U_R(p, r, \delta)$ to emphasize their dependence on the parameters, the set of parameters ω under which this selection rule is efficient is the set of parameters satisfying the incentive constraint (3), namely

$$U_R(p, r, \delta) - U_S(p, r, \delta) \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{4}$$

[10]These payoffs are independent of the workers' labels by symmetry.

We next consider an alternative selection rule for comparison. This selection rule is the same as above, except now that if worker i is selected and delivers a good output, there is inertia: the manager selects the other worker with some probability γ, chosen to be such that the currently selected worker is indifferent between exerting effort and shirking, and reselects worker i otherwise. If this selection rule is efficient, then, by writing $\hat{U}_S$ as the selected worker's continuation payoff and $\hat{U}_R$ as the unselected worker's continuation payoff in each period in the corresponding efficient equilibrium, the payoffs $(\hat{U}_S, \hat{U}_R)$ solve the system of promise-keeping constraints

$$\begin{aligned}\hat{U}_S &= \delta(p\gamma\hat{U}_R + (1 - p\gamma)\hat{U}_S),\\ \hat{U}_R &= (1-\delta)r + \delta(p\gamma\hat{U}_S + (1 - p\gamma)\hat{U}_R).\end{aligned}$$

Writing $\hat{U}_S$ and $\hat{U}_R$ as $\hat{U}_S(p, r, \delta, \gamma)$ and $\hat{U}_R(p, r, \delta, \gamma)$ to emphasize their dependence on the parameters and the transition probability γ, the set of parameters under which this selection rule is efficient is the set of parameters given which there exists $\gamma \in [0, 1]$ satisfying the binding incentive constraint (3), which captures the selected worker's indifference, namely

$$\gamma(\hat{U}_R(p, r, \delta, \gamma) - \hat{U}_S(p, r, \delta, \gamma)) = \frac{(1-\delta)s}{\delta(p-q)}, \qquad \text{for some } \gamma \in [0, 1]. \tag{5}$$

It can be readily verified that the set of parameters ω satisfying (4) is a strict subset of its counterpart satisfying (5): inertia strictly expands the range of parameters achieving efficiency in equilibrium. Intuitively, selection is a punishment, and inertia maximally reduces the worker's continuation payoff whenever selected to create the harshest punishment compatible with efficiency: even after a good output, the currently selected worker might be reselected. This inertia simultaneously and maximally increases the worker's continuation payoff whenever he is not selected to create the strongest reward compatible with efficiency: even after the other worker produces a good output, this worker may remain unselected.

3.2 Rotation

Suppose instead that there are at least three workers. A new problem emerges: when a currently selected worker produces a good output and is then granted rest, whom should the manager pick next among the other workers? The answer to this speaks to rotation. To illustrate, suppose now that there are exactly three workers.

Suppose that the manager chooses an inertial selection rule with uniform shift. Specifically, suppose that the manager initially selects worker 1. In each period on path, if worker $i \in \{1,2,3\}$ is selected, then upon a good output, the manager selects a worker different from i with some probability γ', chosen to be such that the currently selected worker is indifferent between exerting effort and shirking, and continues to select worker i with complementary probability. In the former event, the new worker is drawn uniformly from the other two workers; upon a bad output from worker i, the manager continues to select worker i in the next period. If this selection rule is efficient, then by writing $\hat{U}'_S$ as the selected worker's continuation payoff and $\hat{U}'_R$ as each unselected worker's continuation payoff in each period in the corresponding efficient equilibrium,[11] these payoffs $(\hat{U}'_S, \hat{U}'_R)$ solve the system of promise-keeping constraints

$$\hat{U}'_S = \delta(p\gamma'\hat{U}'_R + (1-p\gamma')\hat{U}'_S),$$
$$\hat{U}'_R = (1-\delta)r + \delta\left(p\frac{\gamma'}{2}\hat{U}'_S + p\frac{\gamma'}{2}\hat{U}'_R + (1-p\gamma')\hat{U}'_R\right).$$

Writing $\hat{U}'_S$ and $\hat{U}'_R$ as $\hat{U}'_S(p,r,\delta,\gamma')$ and $\hat{U}'_R(p,r,\delta,\gamma')$ to emphasize their dependence on the parameters and the transition probability γ', the set of parameters ω under which this selection rule is efficient is the set of parameters given which there exists $\gamma' \in [0,1]$ satisfying the binding incentive constraint (3), namely:

$$\gamma'(\hat{U}'_R(p,r,\delta,\gamma') - \hat{U}'_S(p,r,\delta,\gamma')) = \frac{(1-\delta)s}{\delta(p-q)}, \qquad \text{for some } \gamma' \in [0,1]. \tag{6}$$

Consider an alternative selection rule featuring rotation for comparison. Again, worker 1 is initially selected. In each period following any history, if worker $i \in \{1,2,3\}$ is selected, then upon a good output, the manager selects worker $(i+1) \bmod 3$ with some probability γ'', chosen to be such that the currently selected worker is indifferent between exerting effort and shirking, and continues to select worker i with the complementary probability; upon a bad output, the manager reselects worker i. If this selection rule is efficient, then,

[11] Payoff $\hat{U}'_R$ is the same for the two unselected workers by symmetry.

writing $\hat{U}''_S$ as the selected worker's continuation payoff, $\hat{U}''_{R,1}$ as the continuation payoff of the unselected worker who would be selected next when the currently selected worker is granted rest, and $\hat{U}''_{R,2}$ as the remaining unselected worker's continuation payoff in each period in the corresponding equilibrium, the payoffs $(\hat{U}''_S, \hat{U}''_{R,1}, \hat{U}''_{R,2})$ solve the system of promise-keeping constraints

$$\begin{aligned}\hat{U}''_S &= \delta\left(p\gamma''\hat{U}''_{R,2} + (1-p\gamma'')\hat{U}''_S\right),\\ \hat{U}''_{R,2} &= (1-\delta)r + \delta\left(p\gamma''\hat{U}''_{R,1} + (1-p\gamma'')\hat{U}''_{R,2}\right),\\ \hat{U}''_{R,1} &= (1-\delta)r + \delta\left(p\gamma''\hat{U}''_S + (1-p\gamma'')\hat{U}''_{R,1}\right).\end{aligned}$$

Again writing $\hat{U}''_S$ and $\hat{U}''_{R,2}$ as $\hat{U}'_S(p, r, \delta, \gamma'')$ and $\hat{U}''_{R,2}(p, r, \delta, \gamma'')$ to emphasize their dependence on the parameters and the transition probability γ'', the set of parameters under which this selection rule is efficient is the set of parameters given which there exists $\gamma'' \in [0, 1]$ satisfying the binding incentive constraint (3), namely:

$$\gamma''(\hat{U}''_{R,2}(p, r, \delta, \gamma'') - \hat{U}''_S(p, r, \delta, \gamma'')) = \frac{(1-\delta)s}{\delta(p-q)}, \qquad \text{for some } \gamma'' \in [0, 1]. \tag{7}$$

It can be readily verified that the set of parameters ω satisfying (6) is a strict subset of its counterpart satisfying (7): rotation strictly outperforms uniform shift in attaining efficiency in equilibrium. Intuitively, under the uniform selection rule, each resting worker faces a positive probability of being selected in the next period. This weakens rest as a reward. In contrast, the inertial rotation rule grants a worker who is just given rest after a good output a longer rest time in expectation, leading to stronger effort incentives.

Our two main results, presented next, show that the largest set of parameters under which an efficient equilibrium exists is the set of parameters under which inertial rotation constitutes an efficient equilibrium.

4 Conditions for efficiency

In this section, we present our first main result, characterizing $\Omega^*(n)$, the largest set of parameters given which an efficient equilibrium exists when there are n workers. Let

$V := (V^1, \ldots, V^n)$ denote the (unique) vector solving the system

$$\begin{pmatrix} V^1 \\ V^2 \\ \vdots \\ V^{n-1} \\ V^n \end{pmatrix} = (1-\delta)\begin{pmatrix} 0 \\ r \\ \vdots \\ r \\ r \end{pmatrix} + \delta \left[p \begin{pmatrix} V^2 \\ V^3 \\ \vdots \\ V^n \\ V^1 \end{pmatrix} + (1-p) \begin{pmatrix} V^1 \\ V^2 \\ \vdots \\ V^{n-1} \\ V^n \end{pmatrix} \right]. \tag{8}$$

This system defines $(V^k)_{k=1}^n$ as a linear recurrence that can be solved in closed form, as a function of (n, p, r, δ). Specifically, by writing V^k as $V^k(n, p, r, \delta)$ to emphasize its dependence on the parameters for each k, and by writing $\xi := (1 - \delta(1-p))/(\delta p) > 1$, it holds that

$$V^1(n, p, r, \delta) = \frac{\xi^{n-1} - 1}{\xi^n - 1} r, \tag{9}$$

$$V^k(n, p, r, \delta) = \left(1 - \frac{(\xi - 1)\xi^{k-2}}{\xi^n - 1}\right) r, \qquad \text{for } k = 2, \ldots, n. \tag{10}$$

Our first main result is:

Proposition 1. *Suppose that $n > 1$. It holds that*

$$\Omega^*(n) = \left\{ \omega \in \Omega(n) : V^2(n, p, r, \delta) - V^1(n, p, r, \delta) \geq \frac{(1-\delta)s}{\delta(p-q)} \right\}. \tag{11}$$

Proposition 1 characterizes $\Omega^*(n)$ in terms of the primitives. Because the approach we take to prove this result is unlike existing work (which mainly concerns the case of two workers), in the rest of this section, we sketch a proof of Proposition 1 and discuss its intuition, relegating details to the Appendix. Fix $n > 1$. Let $E \subseteq \mathbf{R}_+^n$ be the set of equilibrium payoff vectors of the n workers,[12] with typical element $U = (U^1, \ldots, U^n)$ where U^i denotes worker i's equilibrium payoff. Let $E^* \subseteq E$ denote the set of efficient equilibrium payoffs (of the workers), attained by each worker exerting effort whenever selected. Therefore

$$E^* = \left\{ U \in E : \sum_{i=1}^n U^i = (n-1)r \right\}, \tag{12}$$

[12] We omit the manager's payoff. In any efficient equilibrium, after any history (on path), the manager's continuation payoff is equal to $pv(\bar{y}) + (1-p)v(\underline{y})$.

because at each history, the selected worker achieves payoff 0 by exerting effort while the remaining $n-1$ workers rest and each achieves payoff r in the equilibrium. By standard arguments, E is compact (see, e.g., Abreu, Pearce and Stacchetti, 1990) and so is E^*. Define

$$\underline{U} := \min_{(U^1,\dots,U^n)\in E^*} U^1. \tag{13}$$

This is the lowest payoff that a worker can obtain in any efficient equilibrium. Next, define

$$\bar{U}^{(1)} := \max_{(\underline{U},U^2,\dots,U^n)\in E^*} U^2.$$

This is the highest payoff that a worker can achieve in any efficient equilibrium conditional on some other worker achieving $\underline{U}$.

The set of parameters $\Omega^*(n)$ supporting efficient equilibria is compact. It is closed, as it is characterized by the worker's incentive constraint (3) whenever selected, which is a weak inequality. It is bounded, as the workers' continuation payoffs are. Consequently, $\Omega^*(n)$ contains its boundary points. Let $\text{bd}(\Omega^*(n))$ denote the boundary of $\Omega^*(n)$. We simultaneously solve for this boundary and properties of efficient equilibria when the parameters lie on this boundary. The boundary then allows us to characterize $\Omega^*(n)$.

Lemma 2 bounds each selected worker's continuation payoff at any history in any efficient equilibrium from below:

Lemma 2. *In any efficient equilibrium, at any history h, for each selected worker i,*

$$U^i(h) \geq \frac{ps}{p-q}. \tag{14}$$

Intuitively, a worker, whenever selected, is willing to exert effort only if he is rewarded with a positive continuation payoff upon a good output. The right side of (14) is the minimum such reward in any efficient equilibrium.

Lemma 3 below shows that for parameters lying on the boundary of $\Omega^*(n)$, in any efficient equilibrium, the difference between the lowest worker's continuation payoff $\underline{U}$ and the highest worker's continuation payoff $\bar{U}^{(1)}$ conditional on some other worker's payoff attaining $\underline{U}$ is equal to the wedge identified in the incentive constraint (3). Moreover, they are the only feasible reward and punishment for each worker.

Lemma 3. *Fix* $\omega \in \mathrm{bd}(\Omega^*(n))$. *It holds that*

$$\bar{U}^{(1)} - \underline{U} = \frac{(1-\delta)s}{\delta(p-q)}. \tag{15}$$

In any efficient equilibrium, for each $i = 1, \ldots, n$, *at each history on path where worker* i *is selected, upon producing a good output, his continuation payoff is equal to* $\bar{U}^{(1)}$; *upon producing a bad output, his continuation payoff is* $\underline{U}$.

Intuitively, because $\omega \in \mathrm{bd}(\Omega^*(n))$, in any efficient equilibrium, the moral hazard problem is so severe that each selected worker is willing to exert effort only because he receives the highest possible continuation payoff $\bar{U}^{(1)}$ upon a good output and receives the lowest counterpart $\underline{U}$ upon a bad output, and even then, this worker is only indifferent between exerting effort and shirking. This observation and the incentive constraint (3) lead to (15).

Lemma 4 next shows that for parameters lying on the boundary of $\Omega^*(n)$, in any efficient equilibrium, the worker's continuation payoff is equal to $\underline{U}$ whenever selected.

Lemma 4. *Fix* $\omega \in \mathrm{bd}(\Omega^*(n))$. *In any efficient equilibrium, for each* $i = 1, \ldots, n$, *at each history* h *where worker* i *is selected, his continuation payoff* $U^i(hi)$ *is equal to* $\underline{U}$.

By Lemma 2, each worker's lowest continuation payoff $\underline{U}$ in any efficient equilibrium is at least $ps/(p-q)$. At the same time, because the parameters lie on the boundary of $\Omega^*(n)$, $\underline{U}$ cannot be strictly higher than $ps/(p-q)$, for otherwise a currently selected worker attaining continuation payoff $\underline{U}$ would have a strict incentive to exert effort, contradicting (15).

Next, given $\underline{U}$ and $\bar{U}^{(1)}$, define, by iteration on $k = 2, \ldots, n-2$,

$$\bar{U}^{(k)} := \max_{(\underline{U}, \bar{U}^{(1)}, \cdots \bar{U}^{(k-1)}, U^{n-k-1}, \ldots, U^n) \in E^*} U^{n-k-1}.$$

This is the best payoff a worker can achieve in any efficient equilibrium conditional on there being k other workers achieving payoffs $\underline{U}, \bar{U}^{(1)}, \ldots \bar{U}^{(k-1)}$. Finally, define

$$\bar{U}^{(n-1)} := (n-1)r - \sum_{k=1}^{n-2} \bar{U}^{(k)} - \underline{U}.$$

This is the only payoff a worker could get in any efficient equilibrium conditional on the other workers achieving $\underline{U}, \bar{U}^{(1)}, \ldots,$ and $\bar{U}^{(n-2)}$ by (12).

By construction, the payoff vector $U := (\underline{U}, \bar{U}^{(1)}, \ldots, \bar{U}^{(n-1)})$ lies in E^*. Lemma 5 shows that this payoff vector is decomposed (in the sense of Abreu et al., 1990) by worker 1 being selected and exerting effort alongside the continuation payoff vector $U' := (\bar{U}^{(1)}, \ldots, \bar{U}^{(n-1)}, \underline{U})$ upon a good output and the payoff vector U upon a bad output. By symmetry across the workers, the payoff vector U' also lies in E^*.

Lemma 5. *Fix $\omega \in \mathrm{bd}(\Omega^*(n))$. It holds that*

$$\begin{pmatrix} \underline{U} \\ \bar{U}^{(1)} \\ \vdots \\ \bar{U}^{(n-1)} \end{pmatrix} = (1-\delta) \begin{pmatrix} 0 \\ r \\ \vdots \\ r \end{pmatrix} + \delta \left[p \begin{pmatrix} \bar{U}^{(1)} \\ \bar{U}^{(2)} \\ \vdots \\ \underline{U} \end{pmatrix} + (1-p) \begin{pmatrix} \underline{U} \\ \bar{U}^{(1)} \\ \vdots \\ \bar{U}^{(n-1)} \end{pmatrix} \right]. \tag{16}$$

Moreover, $(\underline{U}, \bar{U}^{(1)}, \ldots \bar{U}^{(n-1)}) = V$, where V is given by (9) *and* (10)*.*

The first row in (16)—namely, $\underline{U} = \delta(p\bar{U}^{(1)} + (1-p)\underline{U})$—follows from Lemma 3 and Lemma 4: the payoff vector U is necessarily decomposed by worker 1 being selected and exerting effort (and the other workers getting to rest) alongside some efficient continuation payoff vector $w(\bar{y})$ upon a good output and some efficient continuation payoff vector $w(\underline{y})$ upon a bad output, such that worker 1's payoff in $w(\bar{y})$ is equal to $\bar{U}^{(1)}$ and his payoff in $w(\underline{y})$ is equal to $\underline{U}$. Next, in the payoff vector U, for worker 2's payoff to attain $\bar{U}^{(1)}$, his payoff in $w(\bar{y})$ must be at least $\bar{U}^{(2)}$ because $U' \in E^*$ and worker 1's payoff in U' is $\bar{U}^{(1)}$. Because the highest payoff worker 2 can obtain in $w(\bar{y})$ is $\bar{U}^{(2)}$ given that worker 1's payoff in $w(\bar{y})$ is $\bar{U}^{(1)}$, worker 2's payoff in $w(\bar{y})$ is precisely $\bar{U}^{(2)}$. Similarly, worker 2's payoff in $w(\underline{y})$ must be at least $\bar{U}^{(1)}$ because $U \in E^*$ and worker 1's payoff in U is $\underline{U}$. Consequently, the second row in (16) follows—namely, $\bar{U}^{(1)} = (1-\delta)r + \delta(p\bar{U}^{(2)} + (1-p)\bar{U}^{(1)})$. The other rows in (16) follow analogously by iteration. Finally, to see that $(\underline{U}, \bar{U}^{(1)}, \ldots \bar{U}^{(n-1)}) = V$, note that the payoff vector V given by (9) and (10) is a unique solution to the system (16). Moreover, (16) coincides with (8) and V, by definition, uniquely solves (8).

Lemma 5 implies that in the worker's incentive constraint (15) for effort whenever selected on path, $\bar{U}^{(1)} = V^2(n,p,r,\delta)$ and $\underline{U} = V^1(n,p,r,\delta)$. Therefore

$$\mathrm{bd}(\Omega^*(n)) = \left\{ \omega \in \Omega(n) : V^2(n,p,r,\delta) - V^1(n,p,r,\delta) = \frac{(1-\delta)s}{\delta(p-q)} \right\}.$$

Consequently, the set $\Omega^*(n)$ is characterized as in (11), and so Proposition 1 follows.

A notable implication of Lemma 5 is that a selection rule in which the manager selects one of the currently unselected workers with equal probability whenever the currently selected worker produces a good output and is granted rest, such as in De Clippel et al. (2021), fails to sustain an efficient equilibrium for parameters sufficiently close to the boundary of $\Omega^*(n)$, because it gives all unselected workers an identical continuation payoff. This generalizes the discussion at the end of Section 3.

5 Inertial rotation

In this section, we present our second main result, constructing explicitly an efficient equilibrium for each $\omega \in \Omega^*(n)$. We first introduce a definition and some essential notations.

Definition 1 (Rotation)**.** *For any $\alpha \in [0, 1]$, a selection rule is said to be an α-rotating selection rule if it satisfies the following property: in each period, if worker $k = 1, \ldots, n$ is selected, then upon a good output from this worker, the manager selects worker $(k + 1)$ mod n with probability α and continues to select worker k with complementary probability in the next period; upon a bad output instead, the manager continues to select worker k. An α-rotating selection rule is said to be inertial rotation if $\alpha < 1$.*

Intuitively, given a rotating selection rule, the manager cyclically orders the workers. A selected worker remains selected until he delivers a good performance, upon which the manager, with some probability, selects the next worker in the order. Note that for any α, there are multiple α-rotating selection rules, because the initial worker is not uniquely identified. Figure 1 illustrates one such rule in which worker 1 is initially selected.

Proposition 2. *Fix $n > 1$. For any $\omega \in \Omega^*(n)$, there is an efficient equilibrium in which the manager plays an α^*-rotating selection rule for some $\alpha^* \equiv \alpha^*(n, \omega) \in (0, 1]$ and each worker is indifferent between exerting effort and shirking whenever selected.*

The intuition is as follows. For any $\omega \in \Omega^*(n)$, if the inequality in (11) binds, then $\omega \in \mathrm{bd}(\Omega^*(n))$. Given such ω, each worker must be indifferent between exerting effort and shirking under a 1-rotating selection rule whenever he is selected in any efficient equilibrium.

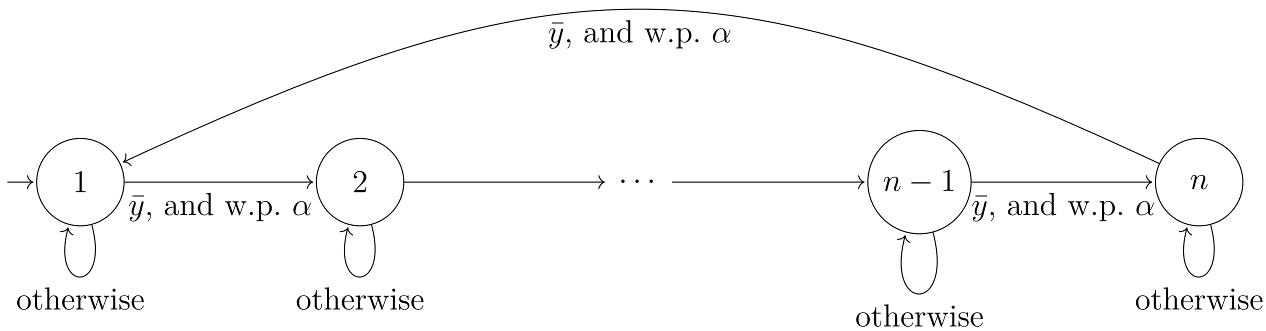

Figure 1: α-rotating selection rule with worker 1 being selected first

If the inequality in (11) holds strictly instead, then there exists a positive probability $\alpha^*(n, \omega)$ of each worker receiving a reward upon a good output that is so low that keeps him indifferent between exerting effort and shirking whenever selected under the $\alpha^*(n, \omega)$-rotating selection rule in any efficient equilibrium. Any such selection rule is therefore efficient. The binding incentive constraint minimizes the continuation payoff of a worker upon producing a bad output subject to efficiency. An α-rotating selection rule, where α minimizes worker transitions without disrupting effort incentives, maximizes the continuation payoff of a worker upon producing a good output when selected and then being allowed to rest, again subject to efficiency. If the selection rule is that such a worker is rewarded more expected rest time than inertial rotation does, then some other worker must be rewarded less expected rest time than rotation does when the former worker is at rest, undermining the latter worker's effort incentives. There is then a range of parameters over which efficiency fails under this alternative selection rule but is attainable by inertial selection.

It is worth remarking that idiosyncrasies, such as workers facing idiosyncratic opportunity costs to exert effort, can be incorporated without affecting our results. Specifically, suppose that in each period, each worker i has a private shirking gain s^i that is independently drawn across time and workers from a distribution G^i on a nondegenerate compact interval $[0, \bar{s}]$. Suppose that efficiency continues to require that a worker exerts effort whenever selected, namely (1) holds with s replaced by $\bar{s}$. Our two main results, Proposition 1 and Proposition 2, readily extend, with s replaced by $\bar{s}$ in (11). If an efficient equilibrium exists, then at every history, the selected worker must have a best reply to exert effort even if his private shirking gain attains $\bar{s}$.

6 Comparative statics

In this section, we discuss the comparative statics concerning the set $\Omega^*(n)$.

Corollary 1. *For each $n > 1$, the left side of* (11) *is strictly increasing in p, r, and δ, and is strictly decreasing in q and s. Moreover, the set $\Omega^*(n)$ is strictly increasing in n and satisfies*

$$\lim_{n\to\infty} \Omega^*(n) = \left\{ (p, q, r, s, \delta) \in \Omega(n) : r - \frac{ps}{p-q} \geq 0 \right\}. \tag{17}$$

For any number of workers $n > 1$, by Proposition 2, any $\alpha^*(n, \omega)$-rotating selection rule is efficient whenever some rule is. Consequently, $\Omega^*(n)$ is equal to the set of parameters in $\Omega(n)$ under which some $\alpha^*(n, \omega)$-rotating selection rule constitutes an efficient equilibrium. Naturally, effort incentives are stronger under any $\alpha^*(n, \omega)$-rotating selection rule when outputs are less noisy—namely, when p is higher or q is lower—the gain r from resting is higher, the workers have a higher discount factor δ, or the gain from shirking s is lower. Next, under any $\alpha^*(n, \omega)$-rotating selection rule, all workers are selected on path. Therefore, given a larger set of workers, each worker's expected rest time conditional on not being selected today is higher. This in turn strengthens effort incentives, leading to a larger range of parameters under which any $\alpha^*(n, \omega)$-rotating selection rule is efficient. Moreover,

$$\lim_{n\to\infty} V^1(n, p, r, \delta) = \frac{1}{\xi} r, \tag{18}$$

$$\text{and} \qquad \lim_{n\to\infty} V^2(n, p, r, \delta) = r, \tag{19}$$

resulting in (17). Intuitively, for any ω on the boundary of $\Omega^*(n)$, $\alpha^*(n, \omega) = 1$. Consequently, the fraction $1/\xi = (\delta p)/(1 - \delta(1 - p))$ is the discounted frequency of a worker being allowed to rest conditional on him being currently selected, giving (18). Further, when there are infinitely many workers, the discounted frequency of a worker being allowed to rest conditional on him currently taking a rest is 1, giving (19).

7 Extensions

In this section, we discuss two extensions of our baseline model.

7.1 Desirable tasks

In this section, we consider a different version of our model where workers prefer to be selected. Suppose instead that in each period, the unselected worker gets payoff normalized to $r < 0$. Here, unlike in our main model, in any efficient equilibrium, being selected (and exerting effort) constitutes a reward whereas being rested is a punishment.

Our analysis above extends, with analogous arguments and minimal modification. Slightly abusing notations, let $V := (V^1, \ldots, V^n)$ denote the (unique) vector solving the system

$$\begin{pmatrix} V^1 \\ V^2 \\ \vdots \\ V^{n-1} \\ V^n \end{pmatrix} = (1-\delta)\begin{pmatrix} 0 \\ r \\ \vdots \\ r \\ r \end{pmatrix} + \delta\left[p\begin{pmatrix} V^1 \\ V^2 \\ \vdots \\ V^{n-1} \\ V^n \end{pmatrix} + (1-p)\begin{pmatrix} V^2 \\ V^3 \\ \vdots \\ V^n \\ V^1 \end{pmatrix}\right]. \tag{20}$$

Like (8), the system (20) defines $(V^k)_{k=1}^n$ as a linear recurrence that can be solved in closed form, as a function of n, p, r, and δ. Specifically, by writing V^k as $V^k(n,p,r,\delta)$ to emphasize its dependence on the parameters for each k, (9) and (10) hold, but now ξ is given instead by $\xi := (1-\delta p)/(\delta(1-p)) > 1$.

Our two main results extend, summarized below in Proposition 3.

Proposition 3. *Suppose that $n > 1$. It holds that*

$$\Omega^*(n) = \left\{\omega \in \Omega(n) : V^1(n,p,r,\delta) - V^2(n,p,r,\delta) \geq \frac{(1-\delta)s}{\delta(p-q)}\right\}. \tag{21}$$

For any $\omega \in \Omega^*(n)$, there is an efficient equilibrium in which the manager plays an α^*-rotating selection rule for some $\alpha^* \equiv \alpha^*(n,\omega) \in (0,1]$ and each worker is indifferent between exerting effort and shirking whenever selected.

We omit the proof because all arguments are analogous. The only difference is that, unlike in (11), in (21) the payoff V^1 captures each worker's highest continuation payoff in any efficient equilibrium whereas V^2 captures the lowest counterpart.

7.2 Selecting teams

In this section, we examine a setting where in each period the manager must select $K \in \{1, \ldots, n\}$ workers. We refer to this subset of selected workers as a team. Once this team is selected, all workers in this team simultaneously and independently choose whether to exert effort or to shirk. If k of them exert effort, then a good output is realized with probability $p_k \in (0, 1)$ and a bad output is realized with complementary probability, where p_k is assumed to be strictly increasing in k. The model is otherwise unchanged. To facilitate a comparison with our main model, we write p_K as p and p_{K-1} as q. As in our main model, if an efficient equilibrium exists, then the number of workers n must be strictly higher than the team size K. Accordingly, we assume that $K < n$. Let $\Omega^*(n, K)$ denote the set of parameters in $\Omega(n)$ under which an efficient equilibrium exists given n workers and given that K workers must be selected in each period.

For concreteness, we assume as in our baseline model that $r > 0$, so that workers prefer to rest. The arguments can be readily adopted to examine the case $r < 0$ where workers dislike being rested as in Section 7.1; We omit this latter case to avoid repetition.

We first generalize Proposition 1 and characterize the set $\Omega^*(n, K)$ for each $n > K$ and $K \geq 1$. Again slightly abusing notations, let $V := (V^0, \ldots, V^0, V^1, \ldots, V^{n-K})$ be the unique solution to the system

$$\begin{pmatrix} V^0 \\ \vdots \\ V^0 \\ V^1 \\ \vdots \\ V^{n-K} \end{pmatrix} = (1-\delta) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ r \\ \vdots \\ r \end{pmatrix} + \delta \left[p \begin{pmatrix} \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} V^i + \frac{(2K-n)^+}{K} V^0 \\ \vdots \\ \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} V^i + \frac{(2K-n)^+}{K} V^0 \\ \mathbf{1}_{\{K+1 > n-K+1\}} V^{K+1} + \mathbf{1}_{\{K+1 \leq n-K+1\}} V^0 \\ \vdots \\ \mathbf{1}_{\{K+n-1 > n-K+1\}} V^{K+n-1} + \mathbf{1}_{\{K+n-1 \leq n-K+1\}} V^0 \end{pmatrix} + (1-p) \begin{pmatrix} V^0 \\ \vdots \\ V^0 \\ V^1 \\ \vdots \\ V^{n-K} \end{pmatrix} \right], \tag{22}$$

where $x^+ \equiv \max(x, 0)$ for any $x \in \mathbf{R}$. Given V, we sometimes write V^i as $V^i(n, K, p, r, \delta)$ for each $i = 0, \ldots, n-K$ to emphasize its dependence on the parameters.

Proposition 4 generalizes Proposition 1:

Proposition 4. *Fix $K \geq 1$ and $n > K$. It holds that*

$$\Omega^*(n,K) = \Bigg\{\omega \in \Omega(n) : \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} V^i(n,K,p,r,\delta) + \frac{(2K-n)^+}{K} V^0(n,K,p,r,\delta) - V^0(n,K,p,r,\delta) \geq \frac{(1-\delta)s}{\delta(p-q)}\Bigg\}. \tag{23}$$

The intuition for Proposition 4 is identical to that of Proposition 1, with one new subtlety. If the number of workers is divisible by the team size K, then our earlier analysis extends in a straightforward manner: the manager partitions the set of workers into teams with equal size and treats each team as a "single worker" as in our main analysis. If the number of workers is not divisible by the team size K, then there are histories on path where a current team of workers produce a good output but not all of them can be rewarded with rest at the same time. In this case, if the parameters belong to the boundary of $\Omega^*(n,K)$, then the workers must be rewarded with equal probabilities to maximize every worker's effort incentives.

To illustrate, consider an example in which five workers are present and in each period, the manager selects two workers, i.e., $n = 5$ and $K = 2$. The system (22) reduces to

$$\begin{pmatrix} V^0 \\ V^0 \\ V^1 \\ V^2 \\ V^3 \end{pmatrix} = (1-\delta)\begin{pmatrix} 0 \\ r \\ \vdots \\ r \\ r \end{pmatrix} + \delta \left[p \begin{pmatrix} \frac{1}{2}V^1 + \frac{1}{2}V^2 \\ \frac{1}{2}V^1 + \frac{1}{2}V^2 \\ V^3 \\ V^0 \\ V^0 \end{pmatrix} + (1-p)\begin{pmatrix} V^0 \\ V^0 \\ V^1 \\ V^2 \\ V^3 \end{pmatrix} \right]. \tag{24}$$

The inequality in (23) reduces to

$$\frac{1}{2}V^1(5,2,p,r,\delta) + \frac{1}{2}V^2(5,2,p,r,\delta) - V^0(5,2,p,r,\delta) \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{25}$$

As in our main model, $(V^0, V^0, V^1, V^2, V^3)$ is a payoff vector in the efficient equilibrium if $\omega \in \text{bd}(\Omega^*(5,2))$, where workers 1 and 2 are currently selected to work. Moreover, when they produce a good output, both must be rewarded the highest possible continuation payoff conditional on two workers being selected; this highest payoff is given by $\frac{1}{2}V^1 + \frac{1}{2}V^2$ in (24). When they produce a bad output, both are punished with the lowest possible continuation payoff conditional on two workers being selected, namely V^0 in (24).

Finally, we generalize Proposition 2, constructing an efficient equilibrium for each $K \geq 1$, $n > K$, and $\omega \in \Omega^*(n, K)$. To this end, we first generalize Definition 1.

Definition 2 (inertial team rotation)**.** *For any $\alpha \in [0, 1]$, a selection rule is said to be an α-rotating team selection rule if it satisfies the following property. In each period, the workers are labeled $w_1, w_2, \ldots, w_n$. Workers w_1 to w_K are selected. Following a good output, with probability α, the manager assigns labels $w_{(k+1) \bmod n}, \ldots, w_{(k+K) \bmod n}$ with equal probabilities among these workers and assigns label $w_{k+K \bmod n}$ to worker w_k for each $k > K$; with complementary probability, the labels are unchanged. Following a bad output, the labels are also unchanged. An α-rotating team selection rule is said to be inertial rotation if $\alpha < 1$.*

Proposition 5 generalizes Proposition 2, with analogous intuition: the rotation among teams are chosen to be as inertial as possible, keeping each worker indifferent between exerting effort and shirking.

Proposition 5. *Fix $n > K$. For any $\omega \in \Omega^*(n, K)$, there is an efficient equilibrium in which the manager plays an α^*-rotating team selection rule for some $\alpha^* \equiv \alpha^*(n, K, \omega) \in (0, 1]$ and each worker is indifferent between exerting effort and shirking whenever selected.*

Therefore, the largest set of parameters given which an efficient equilibrium exists is the set of parameters given which an inertial rotating selection rule is efficient.

8 Concluding remarks

In this paper, we have studied the incentive role of allocating tasks among a group of workers over time. For any group size, we have characterized the largest set of parameters under which efficiency can be sustained in equilibrium and have identified a class of selection rules that achieve efficiency whenever some selection can do so. These rules rotate the worker to be selected in a cyclic order and are maximally inertial: a currently selected worker is reselected with a probability as high as effort incentives allow.

From a technical perspective, although we have developed an approach that enables a systematic analysis of dynamic allocation problems without transfers when there are more than two workers and when more than one worker must be selected in each period, the tractability of our approach relies on ex ante symmetry among workers. Moreover, our model does not entertain the possibility that the manager's information about the workers is incomplete, in which case equilibrium payoffs no longer admit a recursive structure and a different approach is required. We leave these issues for future research.

A Proofs

A.1 Proof of Lemma 1

In any efficient equilibrium σ, at any history h_t where worker i is selected with positive probability, worker i's payoff by exerting effort is given by

$$U^i_{\sigma,f}(h_t i) = (1-\delta) \times 0 + \left(pU^i_{\sigma,f}(h_t i\bar{y}) + (1-p)U^i_{\sigma,f}(h_t i\underline{y})\right).$$

By deviating to shirk, worker i's continuation payoff is

$$\hat{U}^i_{\sigma,f}(h_t i) = (1-\delta) \times s + \left(qU^i_{\sigma,f}(h_t i\bar{y}) + (1-q)U^i_{\sigma,f}(h_t i\underline{y})\right).$$

The one-shot deviation principle requires that $U^i_{\sigma,f}(h_t i) - \hat{U}^i_{\sigma,f}(h_t i) \geq 0$, or equivalently,

$$U^i_{\sigma,f}(h_t i\bar{y}) - U^i_{\sigma,f}(h_t i\underline{y}) \geq \frac{(1-\delta)s}{\delta(p-q)},$$

as was to be shown.

A.2 Proof of Lemma 2

Define

$$\underline{U} := \min_{(U^1,\ldots,U^n)\in E^*} U^1.$$

By Lemma 1, in the equilibrium (σ, f), at history h, the selected worker i's incentive constraint for effort holds:

$$U^i_{\sigma,f}(hi\bar{y}) - U^i_{\sigma,f}(hi\underline{y}) \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{26}$$

Therefore

$$\begin{aligned} U^i_{\sigma,f}(h) &= \delta(pU^i_{\sigma,f}(hi\bar{y}) + (1-p)U^i_{\sigma,f}(hi\underline{y})) \\ &\geq \delta\left(U^i_{\sigma,f}(hi\underline{y}) + \frac{(1-\delta)ps}{\delta(p-q)}\right) \\ &\geq \delta\underline{U} + \frac{(1-\delta)ps}{(p-q)}. \end{aligned}$$

Because σ, h, and i are arbitrarily picked, the above inequality holds when they are picked such that $U^i_{\sigma,f}(h)$ attains $\underline{U}$, giving

$$\underline{U} \geq \delta\underline{U} + \frac{(1-\delta)ps}{(p-q)}.$$

Rearranging gives

$$\underline{U} \geq \frac{ps}{(p-q)}. \tag{27}$$

Consequently,

$$U^i_{\sigma,f}(h) \geq \underline{U} \geq \frac{ps}{(p-q)},$$

as was to be shown.

A.3 Proof of Lemma 3

By Lemma 1, in the equilibrium (σ, f), at history h, upon worker i being selected, by Lemma 2,

$$U^i_{\sigma,f}(hi\underline{y}) \geq \underline{U} \geq \frac{ps}{(p-q)}. \tag{28}$$

Next, at any history h' in the equilibrium, because there must be one worker who is selected and then exerts effort, $U^i_{\sigma,f}(h') \leq \bar{U}^{(1)}$. Consequently,

$$U^i_{\sigma,f}(hi\bar{y}) \leq \bar{U}^{(1)}. \tag{29}$$

Note that (26), (42), and (29) together imply

$$\bar{U}^{(1)} - \underline{U} \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{30}$$

To complete the proof, it suffices to show that this inequality cannot be strict. If it were strict, then by continuity in ω on both sides of (44), there exists an open ball B_ω containing ω such that an efficient selection rule exists given each $\omega' \in B_\omega$, contradicting that $\omega \in \text{bd}(\Omega^*)$.

A.4 Proof of Lemma 4

In the equilibrium, at history h, the selected worker i's continuation payoff $U^i_{\sigma,f}(hi)$ must then be equal to

$$\delta(p\bar{U}^{(1)} + (1-p)\underline{U}). \tag{31}$$

If $U^i_{\sigma,f}(hi\bar{y}) < \bar{U}^{(1)}$, then his incentive constraint to exert effort fails:

$$\begin{aligned} U^i_{\sigma,f}(hi\bar{y}) - U^i_{\sigma,f}(hi\underline{y}) &< \bar{U}^{(1)} - U^i_{\sigma,f}(hi\underline{y}) \\ &\leq \bar{U}^{(1)} - \underline{U} \\ &= \frac{(1-\delta)s}{\delta(p-q)}, \end{aligned}$$

where the last line uses (15). Therefore, (31) and (15) together imply that

$$U^i_{\sigma,f}(hi) = (1-\delta)\frac{ps}{p-q} + \delta\underline{U} \tag{32}$$

Because $U^i_{\sigma,f}(hi) \geq \underline{U}$ by definition of $\underline{U}$,

$$(1-\delta)\frac{ps}{p-q} + \delta\underline{U} \geq \underline{U}.$$

Rearranging,

$$\underline{U} \leq \frac{ps}{p-q}. \tag{33}$$

This and (27) together imply that

$$\underline{U} = \frac{ps}{p-q}. \tag{34}$$

Consequently, by (47),

$$U^i_{\sigma,f}(hi) = (1-\delta)\frac{ps}{p-q} + \delta\frac{ps}{p-q} = \frac{ps}{p-q} = \underline{U}.$$

Therefore

$$\underline{U} = \delta[p\bar{U}^{(1)} + (1-p)\underline{U}]. \tag{35}$$

A.5 Proof of Lemma 5

Define

$$w(y) := \begin{cases} U', & \text{if } y = \bar{y}, \\ U, & \text{if } y = \underline{y}. \end{cases}$$

To prove the lemma, we must show that

$$U = (1-\delta)(0, r, \ldots, r) + \delta[pw(\bar{y}) + (1-p)w(\underline{y})].$$

Let $\mathcal{P}(\mathbf{R}^n)$ denote the set of all subsets of $\mathbf{R}^n$. Let $B : \mathcal{P}(\mathbf{R}^n) \to \mathcal{P}(\mathbf{R}^n)$ denote the generating function (Abreu et al., 1990) so that any set $W \subseteq \mathbf{R}^n$ is said to be self-generating if $W \subseteq B(W)$. For an efficient equilibrium to exists, the set of efficient equilibrium payoffs E^*_f must be self-generating, and therefore $U \in B(E^*_f)$.

For any $\tilde{w}$ that decomposes U, given that the current selected worker 1 exerts effort, it must hold that $\tilde{w}_1(\bar{y}) = U^{(1)}$ and $\tilde{w}_1(\underline{y}) = \underline{U}$ by (35). Moreover, $(\underline{U}, \bar{U}^{(1)}, \ldots, \bar{U}^{(n-1)}) \in E^*_f$ because of symmetry and because $U \in E^*_f$. Therefore, there exists some $\hat{U} = (\underline{U}, \hat{U}^2, \ldots, \hat{U}^n) \in E^*_f$ that is decomposed by the selected worker 1 exerting effort and continuation value w. By definition of $\bar{U}^{(1)}$,

$$\bar{U}^{(1)} > \hat{U}^2. \tag{36}$$

Suppose that there exists a function $w' : \{\bar{y}, \underline{y}\} \to E_f^*$ that decomposes U, given which $w_2'(\bar{y}) \neq \bar{U}^{(2)}$ or $w_2'(\underline{y}) \neq \bar{U}^{(1)}$. Then

$$\begin{aligned}\bar{U}^{(1)} &= (1-\delta)r + \delta[pw_2'(\bar{y}) + (1-p)w_2'(\underline{y})] \\ &< (1-\delta)r + \delta[p\bar{U}^{(2)} + (1-p)\bar{U}^{(1)}] \\ &= (1-\delta)r + \delta[pw_2(\bar{y}) + (1-p)w_2(\bar{y})] = \hat{U}^2,\end{aligned}$$

contradicting (36). Then, by induction, for each $k = 1, \ldots, n-2$, it holds that

$$\bar{U}^{(k)} = (1-\delta)r + \delta[p\bar{U}^{(k+1)} + (1-p)\bar{U}^{(k)}], \qquad \text{for each } k = 1, \ldots, n-2.$$

Finally, because $w(\bar{y}), w(\underline{y}) \in E_f^*$, it follows that

$$\bar{U}^{(n-1)} = (1-\delta)r + \delta[p\underline{U} + (1-p)\bar{U}^{(n-1)}].$$

Consequently, (16) holds, as was to be shown.

A.6 Proof of Proposition 2

For any $\alpha \in (0, 1]$, consider a strategy profile where the manager plays some α-rotating selection rule and the worker exerts effort whenever selected. Under this strategy profile, following any history, let $U_S \equiv U_S(n, p, r, \delta, \alpha)$ denote a worker's continuation payoff upon being selected and for each $k = 0, \ldots, n-2$, let $U_{R,k} \equiv U_{R,k}(n, p, r, \delta, \alpha)$ denote a worker's continuation payoff upon being allowed to rest and conditional on there being k other workers who would be selected before the next time this worker will be selected. The vector

$(U_S, U_{R,n-2}, \ldots, U_{R,0})$ is therefore a unique solution to the system

$$\begin{pmatrix} U_S \\ U_{R,n-2} \\ \vdots \\ U_{R,1} \\ U_{R,0} \end{pmatrix} = (1-\delta)\begin{pmatrix} 0 \\ r \\ \vdots \\ r \\ r \end{pmatrix} + \delta\left[p\alpha \begin{pmatrix} U_{R,n-2} \\ U_{R,n-3} \\ \vdots \\ U_{R,0} \\ U_S \end{pmatrix} + (1-p\alpha)\begin{pmatrix} U_S \\ U_{R,n-2} \\ \vdots \\ U_{R,1} \\ U_{R,0} \end{pmatrix}\right]. \tag{37}$$

Given (16), (37) implies

$$\begin{pmatrix} U_S(n,p,r,\delta,\alpha) \\ U_{R,n-2}(n,p,r,\delta,\alpha) \\ \vdots \\ U_{R,1}(n,p,r,\delta,\alpha) \\ U_{R,0}(n,p,r,\delta,\alpha) \end{pmatrix} = \begin{pmatrix} V^1(n,\alpha p,r,\delta) \\ V^2(n,\alpha p,r,\delta) \\ \vdots \\ V^{n-1}(n,\alpha p,r,\delta) \\ V^n(n,\alpha p,r,\delta) \end{pmatrix}.$$

Moreover,

$$\alpha(U_{R,n-2}(n,p,r,\delta,\alpha) - U_S(n,p,r,\delta,\alpha))\Big|_{\alpha=0} = 0, \tag{38}$$

$$\alpha(U_{R,n-2}(n,p,r,\delta,\alpha) - U_S(n,p,r,\delta,\alpha))\Big|_{\alpha=1} = V^2(n,p,r,\delta) - V^1(n,p,r,\delta) \geq \frac{(1-\delta)s}{\delta(p-q)}, \tag{39}$$

where the inequality uses (11). Also, the expression $\alpha(U_{R,n-2}(n,p,r,\delta,\alpha) - U_S(n,p,r,\delta,\alpha))$ is strictly increasing in α. This is because, by direct computation from (8),

$$\alpha(U_{R,n-2}(n,p,r,\delta,\alpha) - U_S(n,p,r,\delta,\alpha)) = \frac{(1-\delta)r\left(\alpha - \frac{1-\delta}{\delta p\left(\left(\frac{1-\delta}{\alpha\delta p}+1\right)^n - 1\right)}\right)}{1-\delta(1-\alpha p)},$$

so that its derivative with respect to α is positive if and only if

$$(\alpha\delta p + 1 - \delta)^n > (\alpha\delta p)^{n-1}(\alpha\delta p + n(1-\delta)).$$

This inequality holds because

$$
\begin{aligned}
& (\alpha\delta p + 1 - \delta)^n - (\alpha\delta p)^{n-1}(\alpha\delta p + n(1-\delta)) \\
= & \sum_{k=0}^{n} \binom{n}{k} (\alpha\delta p)^{n-k}(1-\delta)^k - (\alpha\delta p)^{n-1}(\alpha\delta p + n(1-\delta)) \\
= & (\alpha\delta p)^n + n(\alpha\delta p)^{n-1}(1-\delta) + \sum_{k=2}^{n} \binom{n}{k} (\alpha\delta p)^{n-k}(1-\delta)^k - (\alpha\delta p)^{n-1}(\alpha\delta p + n(1-\delta)) \\
= & \sum_{k=2}^{n} \binom{n}{k} (\alpha\delta p)^{n-k}(1-\delta)^k \\
> & \, 0,
\end{aligned}
$$

where the second line uses the binomial theorem. Consequently, by (51) and (52), and by monotonicity of $\alpha(U_{R,n-2}(n,p,r,\delta,\alpha) - U_S(n,p,r,\delta,\alpha))$ in α, there exists $\alpha^* \equiv \alpha^*(n,\omega) \in (0,1]$ such that

$$\alpha^*(U_{R,n-2}(n,p,r,\delta,\alpha^*) - U_S(\alpha^*(n,p,r,\delta,\omega))) = \frac{(1-\delta)s}{\delta(p-q)}. \tag{40}$$

It then follows that any $\alpha^*(n,\omega)$-rotating selection rule is efficient: given such selection rule, an efficient equilibrium exists because each selected worker's incentive constraint for effort in this equilibrium is given by (40) and therefore holds.

A.7 Proof of Proposition 4

Define

$$\bar{U}^{(1)} := \max_{(\underline{U},\ldots\underline{U},U^{K+1},\ldots,U^n)\in E^*} U^{K+1}.$$

This is the highest payoff that a worker can achieve in any efficient equilibrium conditional on K workers achieving $\underline{U}$. Next, by iteration on $k = 2,\ldots,n-K-1$,

$$\bar{U}^{(2)} := \max_{(\underline{U},\ldots\underline{U},\bar{U}^{(1)},\ldots,\bar{U}^{(k)},U^{n-k-K},\ldots,U^n)\in E^*} U^{n-k-K}.$$

This is the best payoff a worker can achieve in any efficient equilibrium conditional on there being K other workers achieving $\underline{U}$, and also others achieving $\bar{U}^{(1)}, \dots, \bar{U}^{(k)}$. Finally, define

$$\bar{U}^{(n-K)} := (n-K)r - K\underline{U} - \sum_{k=1}^{n-K-1} U^{(k)}.$$

This is the only payoff a worker could get in any efficient equilibrium conditional on there being K other workers achieving $\underline{U}$, K achieving $\bar{U}^{(1)}$, K achieving $\bar{U}^{(2)}$,..., and K achieving $\bar{U}^{m-1}$. To prove the necessity of (23), we begin by characterizing the boundary of $\Omega^*(n, K)$, the largest set of parameters given which an efficient selection rule exists. We denote this boundary by $\text{bd}(\Omega^*(n, K))$. Fix some $\omega \in \text{bd}(\Omega^*(n, K))$ and an efficient equilibrium (σ, f).

Lemma 6. *In this equilibrium, for each $i = 1, \dots, n$, at each history h where worker i is selected, it holds that*

$$\sum_{i=1}^{\min(K,n-K)} \frac{1}{K}\bar{U}^{(i)}(n, K, p, r, \delta) + \frac{(2K-n)^+}{K}\underline{U} - \underline{U} = \frac{(1-\delta)s}{\delta(p-q)}. \tag{41}$$

Proof of Lemma 6. By Lemma 1 and Lemma 2, in the equilibrium (σ, f), at history h, upon worker i being selected,

$$U^i_{\sigma,f}(hi\underline{y}) \geq \underline{U} \geq \frac{ps}{(p-q)}. \tag{42}$$

We argue that there is a uniform bound U^* for each such worker i,

$$U^i_{\sigma,f}(hi\bar{y}) \leq U^*. \tag{43}$$

Suppose that there exists a worker j who is selected at history h, given which (43) fails. Then, there exists another worker k who is selected at history h, given which (43) holds strictly. But then $\omega \notin \text{bd}(\Omega^*(n, K))$. This is because the manager, by randomizing the continuation for worker k and worker j, strictly increases the value of $U^k_{\sigma,f}(hk\bar{y}) - U^k_{\sigma,f}(hj\underline{y})$, so that efficiency can be attained for a larger set of parameters. Therefore, by (26), (42), and (43),

$$\hat{U} - \underline{U} \geq \frac{(1-\delta)s}{\delta(p-q)}. \tag{44}$$

This inequality cannot be strict. If it were strict, then by continuity in ω on both sides of (44), there exists an open ball B_ω containing ω such that an efficient selection rule exists given each $\omega' \in B_\omega$, contradicting that $\omega \in \text{bd}(\Omega^*)$. Finally, because $U^i_{\sigma,f}(hi\bar{y}) = \hat{U}$ for all workers i that are selected and $\omega \in \text{bd}(\Omega^*(n,K))$, because there are K workers who are selected in each period, by definition of $\bar{U}^{(1)}, \bar{U}^{(2)}, \ldots,$

$$U^* = \sum_{i=1}^{\min(K,n-K)} \frac{1}{K}\bar{U}^{(i)}(n,K,p,r,\delta) + \frac{(2K-n)^+}{K}\underline{U}. \tag{45}$$

This proves (41). ■

Lemma 7. *In this equilibrium, for each $i = 1, \ldots, n$, at each history h where worker i is selected, his continuation payoff $U^i_{\sigma,f}(hi)$ is equal to $\underline{U}$. At this history, upon producing a good output, his continuation payoff is equal to* (45)*; upon producing a bad output, his continuation payoff remains to be $\underline{U}$.*

Proof of Lemma 7. By Lemma 6, in the equilibrium, at history h, each selected worker i's continuation payoff $U^i_{\sigma,f}(hi)$ must be equal to

$$\delta(pU^* + (1-p)\underline{U}). \tag{46}$$

If $U^i_{\sigma,f}(hi\bar{y}) < U^*$, then his incentive constraint to exert effort fails:

$$\begin{aligned} U^i_{\sigma,f}(hi\bar{y}) - U^i_{\sigma,f}(hi\underline{y}) &< U^* - U^i_{\sigma,f}(hi\underline{y}) \\ &\le U^* - \underline{U} \\ &= \frac{(1-\delta)s}{\delta(p-q)}, \end{aligned}$$

where the last line uses (41). Similarly, if $U^i_{\sigma,f}(hi\underline{y}) > U^*$, then his incentive constraint to exert effort also fails. Therefore, (46) and (41) together imply that

$$U^i_{\sigma,f}(hi) = (1-\delta)\frac{ps}{p-q} + \delta\underline{U}. \tag{47}$$

Because $U^i_{\sigma,f}(hi) \geq \underline{U}$ by definition of $\underline{U}$,

$$(1-\delta)\frac{ps}{p-q} + \delta\underline{U} \geq \underline{U}.$$

Rearranging, (33) holds. This and (27) together imply that (34) holds. Thus, by (47),

$$\begin{aligned} U^i_{\sigma,f}(hi) &= (1-\delta)\frac{ps}{p-q} + \delta\frac{ps}{p-q} \\ &= \frac{ps}{p-q} \\ &= \underline{U}. \end{aligned}$$

Therefore

$$\underline{U} = \delta[pU^* + (1-p)\underline{U}], \tag{48}$$

as was to be shown. ■

The payoff vector $U := (\underline{U}, \ldots, \underline{U}, \bar{U}^{(1)}, \ldots, \bar{U}^{(n-K-1)})$ lies in the set of efficient equilibrium payoffs E^*. This set E^* must be self-generating. In particular, Lemma 5 shows that this payoff vector is decomposed by workers 1 to K being selected and exerting effort alongside continuation payoffs $U' := (U^*, \ldots, U^*, \bar{U}^{(K+1)}, \ldots, \bar{U}^{(n-K-1)}, \underline{U}, \ldots, \underline{U})$ upon a good output and U upon a bad output. By symmetry across the workers, U' also lies in E^*.

Lemma 8. *For $\omega \in \mathrm{bd}(\Omega^*(n, K))$, it holds that*

$$\begin{pmatrix} \underline{U} \\ \vdots \\ \underline{U} \\ \bar{U}^{(1)} \\ \vdots \\ \bar{U}^{(n-K)} \end{pmatrix} = (1-\delta) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ r \\ \vdots \\ r \end{pmatrix}$$

$$+ \delta \left[p \begin{pmatrix} \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} \bar{U}^{(i)} + \frac{(2K-n)^+}{K} \underline{U} \\ \vdots \\ \sum_{i=1}^{\min(2K,n-K)} \frac{1}{K} \bar{U}^{(i)} + \frac{(2K-n)^+}{K} \underline{U} \\ \mathbf{1}_{\{K+1>n-K+1\}} \bar{U}^{(K+1)} + \mathbf{1}_{\{K+1 \le n-K+1\}} \underline{U} \\ \vdots \\ \mathbf{1}_{\{K+n-1>n-K+1\}} \bar{U}^{(K+n-1)} + \mathbf{1}_{\{K+n-1 \le n-K+1\}} \underline{U} \end{pmatrix} + (1-p) \begin{pmatrix} \underline{U} \\ \vdots \\ \underline{U} \\ \bar{U}^{(1)} \\ \vdots \\ \bar{U}^{(n-K)} \end{pmatrix} \right]. \tag{49}$$

Proof of Lemma 8. Define

$$w(y) := \begin{cases} U', & \text{if } y = \bar{y}, \\ U, & \text{if } y = \underline{y}. \end{cases}$$

To prove the lemma, we must show that

$$U = (1-\delta)(0, r, \dots, r) + \delta[pw(\bar{y}) + (1-p)w(\underline{y})].$$

Let $\mathcal{P}(\mathbf{R}^n)$ denote the set of all subsets of $\mathbf{R}^n$. Let $B : \mathcal{P}(\mathbf{R}^n) \to \mathcal{P}(\mathbf{R}^n)$ denote the generating function (Abreu et al., 1990) so that any set $W \subseteq \mathbf{R}^n$ is said to be self-generating if $W \subseteq B(W)$. For an efficient equilibrium to exists, the set of efficient equilibrium payoffs E^* must be self-generating, and therefore $U \in B(E^*)$. Given that the current selected worker $k = 1, \dots, K$ exerts effort, it must hold that $\tilde{w}_k(\bar{y}) = U^*$ and $\tilde{w}_1(\underline{y}) = \underline{U}$ by (48). For each worker $k > K$, by definition of $\bar{U}^{(k)}$, $w_k(\bar{y}) = \mathbf{1}_{\{K+1>n-K+1\}} \bar{U}^{(K+1)} + \mathbf{1}_{\{K+1 \le n-K+1\}} \underline{U}$ and $w_k(\underline{y}) = \bar{U}^{(k)}$, for the same reason as in the proof of Lemma 8. Consequently, (49) holds, as was to be shown. ■

Lemma 9. *It holds that* $(\underline{U}, \ldots, \underline{U}, \bar{U}^{(1)}, \ldots \bar{U}^{(n-K)}) = V$*, where* V *solves* (22).

Proof. This is because (49) coincides with (22) and the solution to these systems is unique.

■

Finally, for any $\omega \in \Omega^*(n, K)$ and any efficient equilibrium (σ, f), for any history h and any selected worker i at this history, $U^i_{\sigma,f}(hi\bar{y}) \leq \bar{U}^{(1)} = V^2(n, p, r, \delta)$ and $U^i_{\sigma,f}(hi\underline{y}) \geq \underline{U} = V^1(n, p, r, \delta)$. These inequalities, alongside the fact that the incentive constraint (26) for effort at each history h for the selected worker must hold in the efficient equilibrium, imply that (23) necessarily holds, as was to be shown.

It remains to show that (23) is also sufficient for an efficient selection rule to exist.

Lemma 10. *Suppose that* (23) *holds. Then there exists* $\alpha^* \equiv \alpha^*(n, K, \omega) \in (0, 1]$ *such that any* $\alpha^*(n, K, \omega)$*-rotating selection rule is efficient.*

Proof of Lemma 10. For any $\alpha \in (0, 1]$, consider a strategy profile where the manager plays some generalized α-rotating selection rule and the workers exert effort whenever selected. Under this rotation rule and the strategy profile, following any history, let $U_S \equiv U_S(n, p, r, \delta, \alpha)$ denote a worker's continuation payoff upon being selected and for each $k = 0, \ldots, n-2$, let $U_{R,k} \equiv U_{R,k}(n, p, r, \delta, \alpha)$ denote a worker's continuation payoff upon being allowed to rest and conditional on there being k other workers who would be selected before the next time this worker will be selected. The vector $(U_S, U_{R,n-2}, \ldots, U_{R,0})$ is therefore a unique solution to the system

$$
\begin{pmatrix} U_S \\ \vdots \\ U_S \\ U_{R,1} \\ \vdots \\ U_{R,n-K} \end{pmatrix} = (1-\delta) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ r \\ \vdots \\ r \end{pmatrix}
+ \delta \left[p\alpha \begin{pmatrix} \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} U_{R,i} + \frac{(2K-n)^+}{K} U_S \\ \vdots \\ \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} U_{R,i} + \frac{(2K-n)^+}{K} U_S \\ \mathbf{1}_{\{K+1>n-K+1\}} U_{R,K+1} + \mathbf{1}_{\{K+1 \leq n-K+1\}} U_S \\ \dots \\ \mathbf{1}_{\{K+n-1>n-K+1\}} U_{R,K+n+1} + \mathbf{1}_{\{K+n-1 \leq n-K+1\}} U_S \end{pmatrix} + (1-p\alpha) \begin{pmatrix} U_S \\ \vdots \\ U_S \\ U_{R,1} \\ \vdots \\ U_{R,n-K} \end{pmatrix} \right]. \tag{50}
$$

Given (49), (50) implies

$$
\begin{pmatrix} U_S(n,K,p,r,\delta,\alpha) \\ \vdots \\ U_S(n,K,p,r,\delta,\alpha) \\ U_{R,1}(n,K,p,r,\delta,\alpha) \\ \vdots \\ U_{R,n-K}(n,K,p,r,\delta,\alpha) \end{pmatrix} = \begin{pmatrix} V^0(n,K,\alpha p,r,\delta) \\ \dots \\ V^0(n,K,\alpha p,r,\delta) \\ V^1(n,K,\alpha p,r,\delta) \\ \vdots \\ V^{n-K}(n,\alpha p,r,\delta) \end{pmatrix}.
$$

Moreover,

$$
\alpha(U_{R,n-2}(n,K,p,r,\delta,\alpha) - U_S(n,K,p,r,\delta,\alpha))\Big|_{\alpha=0} = 0, \tag{51}
$$

$$
\begin{aligned}
&\alpha(U_{R,n-2}(n,K,p,r,\delta,\alpha) - U_S(n,K,p,r,\delta,\alpha))\Big|_{\alpha=1} \\
&= \sum_{i=1}^{\min(K,n-K)} \frac{1}{K} V^i(n,K,p,r,\delta) + \frac{(2K-n)^+}{K} V^0(n,K,p,r,\delta) - V^1(n,K,p,r,\delta) \geq \frac{(1-\delta)s}{\delta(p-q)},
\end{aligned} \tag{52}
$$

where the inequality uses (11). By continuity of $\alpha(U_{R,n-2}(n,K,p,r,\delta,\alpha)-U_S(n,K,p,r,\delta,\alpha))$ in α, there exists $\alpha^* \equiv \alpha^*(n,K,\omega) \in (0,1]$ such that

$$\alpha^*(U_{R,n-2}(n,K,p,r,\delta,\alpha^*) - U_S(n,K,p,r,\delta,\alpha^*)) = \frac{(1-\delta)s}{\delta(p-q)}. \tag{53}$$

It then follows that any $\alpha^*(n,K,\omega)$-rotating selection rule is efficient: given such selection rule, an efficient equilibrium exists because each selected worker's incentive constraint for effort in this equilibrium is given by (53) and therefore holds. ■

References

Abreu, D., Pearce, D. and Stacchetti, E. (1990). Toward a Theory of Discounted Repeated Games with Imperfect Monitoring, *Econometrica: Journal of the Econometric Society* pp. 1041–1063. (pp. 13, 15, 27, and 34).

Andrews, I. and Barron, D. (2016). The Allocation of Future Business: Dynamic Relational Contracts with Multiple Agents, *American Economic Review* **106**(9): 2742–2759. (p. 4).

Athey, S. and Bagwell, K. (2001). Optimal Collusion with Private Information, *RAND Journal of Economics* pp. 428–465. (p. 4).

Board, S. (2011). Relational Contracts and the Value of Loyalty, *American Economic Review* **101**(7): 3349–3367. (p. 4).

De Clippel, G., Eliaz, K., Fershtman, D. and Rozen, K. (2021). On Selecting The Right Agent, *Theoretical Economics* **16**(2): 381–402. (pp. 2, 4, and 16).

Ekmekci, M. (2011). Sustainable Reputations with Rating Systems, *Journal of Economic Theory* **146**(2): 479–503. (p. 4).

Ely, J. C., Georgiadis, G. and Rayo, L. (2025). Feedback Design in Dynamic Moral Hazard, *Econometrica* **93**(2): 597–621. (p. 4).

Fong, Y.-F. and Li, J. (2016). Information Revelation in Relational Contracts, *The Review of Economic Studies* **84**(1): 277–299. (p. 4).

Fudenberg, D. and Levine, D. K. (1994). Efficiency and Observability with Long-Run and Short-Run Players, *Journal of Economic Theory* **62**(1): 103–135. (p. 6).

Halac, M. and Prat, A. (2016). Managerial Attention and Worker Performance, *American Economic Review* **106**(10): 3104–3132. (p. 4).

Hörner, J. and Lambert, N. (2021). Motivational Ratings, *The Review of Economic Studies* . (p. 4).

Knauth, P. and Hornberger, S. (2003). Preventive and Compensatory Measures for Shift Workers, *Occupational Medicine* **53**(2): 109–116. (p. 3).

Li, J., Matouschek, N. and Powell, M. (2017). Power Dynamics in Organizations, *American Economic Journal: Microeconomics* **9**(1): 217–241. (p. 4).

Lipnowski, E. and Ramos, J. (2020). Repeated Delegation, *Journal of Economic Theory* **188**: 105040. (p. 4).

McHugh, M., Farley, D. and Rivera, A. S. (2020). A Qualitative Exploration of Shift Work and Employee Well-being in the US Manufacturing Environment, *Journal of Occupational and Environmental Medicine* **62**(4): 303–306. (p. 3).

Vong, A. (2025a). Mediation and Worker Performance, *Working paper, National University of Singapore* . (p. 4).

Vong, A. (2025b). Reputation and Efficiency: Information Design, *American Economic Journal: Microeconomics* . (p. 4).

Winter, E. (2004). Incentives and Discrimination, *American Economic Review* **94**(3): 764–773. (p. 3).